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KNOWLEDGE ACQUISITION VIA THREE LEARNING PROCESSES IN ENTERPRISE INFORMATION PORTALS: LEARNING-BY-INVESTMENT, LEARNING-BY-DOING, AND LEARNING-FROM-OTHERS[1]

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Abstract

An enterprise information portal (EIP) is viewed as a knowledge community. Activity theory provides a framework to study such a community: members of an EIP conduct specific tasks that are assigned through a division of labor. Each member of an enterprise information portal can undergo three distinct types of learning processes: learning-by-investment, learning-by-doing, and learning-from-others. Through these three types of learning processes, each member achieves specialized knowledge that is related to his or her own task. Cumulative knowledge resulting from the learning processes is considered in terms of two distinct attributes: depth and breadth of knowledge. This paper formulates a mathematical model and defines the goal of an EIP member as maximizing

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[1] V. Sambamurthy and Mani Subramani were the accepting senior editors for this paper.

the net benefits of knowledge resulting from individual investment and effort. Numerical examples are provided to analyze patterns of optimal investment and effort plans as well as the resulting accumulated knowledge. The results provide useful managerial implications. In business conditions characterized by high interest rates or high internal rate of returns, it is preferable for members to delay spending their resources for learning. Intensive investment and efforts to obtain knowledge are preferable when the discount rate of costs is high, when knowledge is durable, when the value of knowledge is high, when the initial level of knowledge is high, when the productivity of the learning process is high, and when sufficient knowledge is transferred from other members. On the other hand, the size of the EIP has a positive or negative effect depending on the attribute of knowledge and the productivity of learning processes. Further properties of the optimal decisions and learning processes are analyzed and discussed.

Keywords: Knowledge management, enterprise information portals, learning, activity theory

Introduction

There has been extensive research on knowledge management related to information technology (Alavi and Leidner 2001; Becerra-Fernandez and Sabherwal 2001; Earl 2001; Grover and Davenport 2001), but relatively little attention has been given to learning processes that form the critical part of knowledge acquisition in knowledge portals. This study focuses on knowledge acquisition processes and the individual user's investment strategy to obtain the optimal benefit from specialized knowledge in enterprise information portals (EIP). An EIP, in this study, is defined as a knowledge portal whose main function is to assist its members in obtaining specialized knowledge through various learning processes (Dias 2001; Mack et al. 2001). Further, an EIP can be considered to be a knowledge community (Bakos and Brynjolfsson 1997; Strader et al. 1998) that is composed of employees in a single company or in multiple companies that have relationships with each other. This paper utilizes an activity theory framework (Kim et al. 2002) to put forward the concept that members of the EIP conduct specific tasks that are assigned through a division of labor. The paper proposes a model wherein, through three types of learning processes (i.e., learning-by-investment, learning-by-doing, and learning-from-others), members obtain the optimal amount of specialized knowledge that is related to their own tasks. The three learning processes are modeled to have different impacts on depth and breadth of knowledge. Each member of an EIP decides optimal investments and efforts in each learning process, taking account of various conditions such as discount rate of cost, internal rate of return, and decay rate of knowledge. This study analyzes a mathematical model to explain the impact of each condition on individual decisions on the investment in three learning processes (Rao et al. 1995).

This research is expected to contribute to the literature in two ways. First, solutions of the mathematical model identify possible optimal individual decisions for obtaining the maximum benefit of knowledge under the given environmental conditions. The optimal solutions drawn from the analysis may explicitly provide individuals with proper guidance to knowledge investment. Second, the proposed model refers to an individual. However, the conclusions are equally applicable to a firm or a group as knowledge is a non-rival good in that there is no loss in sharing (Foray 2004). The findings of this study suggest appropriate guidelines for an EIP design policy to facilitate knowledge activities of members and achieve organizational effectiveness.

EIP as a Knowledge Portal and Learning Processes

Enterprise information portals are of multiple forms, ranging from Internet-based data management tools that bring visibility to previously dormant data so that their users can compare, analyze, and share enterprise information (Kim et al. 2002) to a knowledge portal, which enables its users to obtain specialized knowledge that is related to their specific tasks (Dias 2001; Mack et al. 2001). The EIP in this paper is assumed to be one that delivers information to users who constitute a knowledge community (Chan and Chung 2002; Goff 2001). Knowledge is a good that is often cumulative. Many types of knowledge such as databases, research tools, or generic knowledge or even physics are strongly cumulative, while others like songs and poems are non-cumulative.

> Knowledge production has therefore the potential to create a combinatorial explosion…. This is a good which can be used infinitely to produce other knowledge which in turn is non rival and cumulative (Foray 2004).

Kim et al. (2002) explain knowledge management activities in the context of EIP from an activity theory perspective. In their model, each member of an EIP acquires or transfers knowledge through social interaction based on communities of practice. Each member of an EIP has specialized knowledge to accomplish tasks at hand in the organizational structure, resulting in *division of labor* (Brown and Duguid 1998). Each member of an EIP also exchanges knowledge among members. This study applies the same concept of the EIP to explain learning processes by which members achieve specialized knowledge associated with tasks.

Activity theory explains the transformation process of an object to outcomes in terms of three core components (actor, community, and object), three mediating components (tool, rule, and division of labor), and their relationships (Bellamy 1996; Hasan and Gould 2001; Kuutti 1996). Within the activity theory perspective, an individual member participating in the EIP conducts three activities for knowledge acquisition. First, as a member of the EIP, the member needs to keep *making investments in knowledge* to increase his or her level of knowledge. A certain amount of resources and time must be consumed to search for new skills and techniques. The outcomes of this learning process are specialized knowledge, which is associated with the particular task assigned by the EIP to the member, and the acquired knowledge, which increases efficiency in accomplishing tasks. An actor acquires necessary knowledge regarding his or her division of labor for performing the task or object. In this study, this process is referred to as *learning-by-investment*.

Second, the member should *accumulate specialized knowledge* by doing tasks in the EIP. Each member of the EIP has specific tasks within the EIP organization. While the member carries out assigned tasks, he or she should keep accumulating specialized knowledge for these tasks. Members learn from experience with working on their own tasks in the EIP, and they accumulate specialized knowledge by receiving feedback from their own experiences. It is also important for the member to prevent formerly accumulated knowledge from being lost. Even though the learning-by-doing process may be considered routine, all members of the EIP are still obligated to make efforts at accumulating and maintaining specialized knowledge that will allow them to work more efficiently than before. From the activity perspective, the transformation process of object (task) to outcome is the key component for the second process of knowledge acquisition. The gap between actual outcome and expected outcome that occurred in the transformation process is added to the initial knowledge through a *feedback* process. The transformation process and the feedback process constitute *learning-by-doing*.

Finally, every member of the EIP is committed to transfer knowledge to the other members and receive it from others. In general, the ultimate goal of the EIP is to integrate knowledge that was previously distributed to individuals. Only if members of the EIP actively transfer their own knowledge to other members and receive knowledge from others can the EIP become a true knowledge portal instead of a simple Web site that posts information. Knowledge transfer is mostly carried out through *communication between members*. In fact, the EIP supports direct and indirect communication between its members by using advanced technologies such as electronic

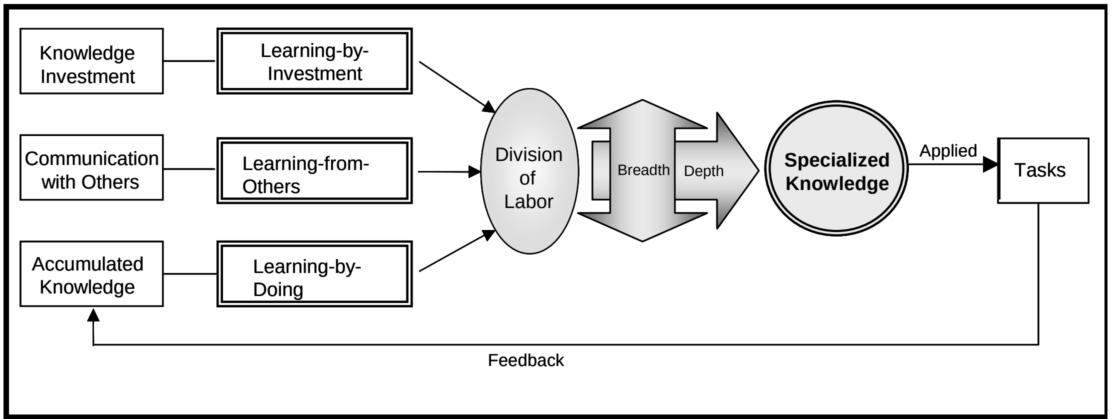

Figure 1. Activities and Learning Processes in the EIP

mail, groupware, etc. Knowledge transfer is not limited to members who conduct similar tasks within the same division, but it also occurs between members who are charged with different tasks and who *learn from* each *other*. In the activity perspective, communities of practice are involved in this type of knowledge acquisition, which is called *learning-from-others*.

These three distinct activities, (1) *learning-by-investment,* (2) *learning-by-doing,* and (3) *learning-from-others*, become the major sources of learning where each member of the EIP acquires *specialized knowledge*.

The outcome of learning processes is measured by the accumulated task-related knowledge and it is evaluated based on two distinct attributes of knowledge: *depth* and *breadth*. The depth of knowledge indicates how much knowledge is focused and pertinent in its content. Due to its connection with the specific task, the depth of knowledge is closely related to the level of specialization in tasks, which are assigned to members. Knowledge is also measured in terms of its breadth, which represents the diversity of knowledge across members (Turner et al. 2002). The division of labor affects both depth and breadth of accumulated knowledge. When the division of labor is high, members of that EIP may improve the depth of knowledge through learning-by-doing. However, the high level of division of labor negatively influences the breadth of knowledge, because it limits the diversity of an individual member's task and vision. When it is required to enhance members' knowledge in terms of its breadth, the EIP should expedite different types of learning processes, which are learning-by-investment and learning-from-others between distinct divisions. Figure 1 depicts the three learning processes.

The levels of accumulated knowledge through the three learning processes are influenced by a variety of factors including decay rate of knowledge (Pakes and Schankerman 1979), discount rate of cost (Dorroh et al. 1994), the number of participants joining knowledge creation and transfer activities (De Liso et al. 2001), initial level of knowledge of the member, the amount of knowledge transferred from other members, and the productivity of the learning-by-doing process. We discuss these factors in the research model section.

Background and Research Model

The focus of the current paper is the three learning processes for knowledge accumulation of EIP members and the strategy of knowledge investment and efforts over time. Accordingly, this study presents the optimization problem that maximizes the total profit, which consists of the revenue from accumulated knowledge and costs incurred by the three learning processes. In this section, the effect of the three learning processes on accumulated knowledge and the factors (i.e., discount rate for knowledge and cost, the number of members, etc.) influencing revenue and costs are discussed.

Three Learning Processes

Learning-by-Investment and Learning-by-Doing

Learning-by-investment and learning-by-doing represent the comprehensive knowledge acquisition processes that each member follows while participating in the EIP. Prior research has focused to an extent on learning-by-investment and learning-by-doing. Ba et al. (2001) address an organization's investment in internal knowledge. The objective of their model is to maximize the net surplus resulting from trading knowledge bundles, where knowledge providers offer a bundle of knowledge and auction bids are placed by knowledge consumers who desire it. In other studies in economic literature, learning is considered a production experience measured as accumulated output relative to inputs of labor and investment (Arrow et al. 1961). Dorroh et al. (1986, 1994) develop an economic model explaining the learning process that occurs during production. Their model addresses not only the learning-by-doing effect but also the learning-by-investment effect. They define cumulative knowledge as a direct output resulting from investment as well as a byproduct of production experience. The model developed by De Liso et al. (2001) offers an essential background based upon which we represent the relationship between learning-by-doing processes and division of labor. They assume that the level of the division of labor in the organization grows with more labor input. To determine the influence of division of labor on learning effect, they characterize the Cobb-Douglas production function by formulating the returns to scale of the learning-by-doing effect to be an increasing value as division of labor intensifies.

Following the above literature, we adopt a similar type of Cobb-Douglas production function to represent learning-by-investment and learning-by-doing effects. The fundamental difference between this study and the studies by Dorroh et al. is that the return to scale, which indicates the rate of producing cumulative knowledge, is not merely a given parameter as they assume, but rather is dependent on the level of division of labor in the EIP organization. We apply the logistic function to define the returns to scale of the learning process as a function of division of labor. Another difference between the model in this paper and the models used in prior research is in the measurement of knowledge. De Liso et al. perceive knowledge as a one-dimensional measure and they assume that knowledge simply increases with higher division of labor. The model herein, however, differentiates the influence of the division of labor in terms of two distinct measures of knowledge, depth and breadth, based on the claim that division of labor has different impacts on knowledge depending on its distinct attributes.

The level of division of labor has a significant impact on an individual's knowledge acquisition. When the level of division of labor is high, the task of each individual member becomes more specialized and consequently the members can enhance the depth of knowledge by focusing on the limited area of their specialized tasks. On the other hand, since the focus of each individual member's work becomes narrower within a limited area, members of an EIP having a high level of division of labor may lose chances to improve the breadth of knowledge. Division of labor is positively correlated with the depth of knowledge, but is negatively correlated with the breadth of knowledge. Therefore, this study formulates the return to scale of the learning process in terms of two separate functions that show distinct influences of division of labor on different attributes of knowledge.

Learning-from-Others

In contrast to the previous two types of learning processes, the learning-from-others effect has not been well studied as a key subject in the information systems area. What is learned from other members of an organization is always critical in terms of its impact on the knowledge acquisition of individual members as well as on the organization. The learning-from-others effect is mostly carried out through formal or informal communication among members of the same or different organizations. The main role of information technology is to support communication and link individuals within and between functions and divisions through database repositories, or electronic mail, etc. (Dewett and Jones 2001).

There exist some studies that address learning processes that are similar to the learning-from-others effect. In illustrating the diffusion of experience based on the ecology of learning, Herriott et al. (1985) describe how one actor can supplement learning from direct experience through diffusion of experience by copying others. Their model represents this learning effect by hypothesizing that budget allocation, competences, and goals related to specific activities are dependent on other actors' behaviors as well as on one's own experience. Chakrabarti and Roll (1999) investigate circumscribed learning (i.e., deriving information by simply observing the actions of other traders).

The proposed model in this paper is based on the fact that a member of the EIP actively obtains knowledge by communicating with other members, not just by observing or copying others. Members are able to communicate with each other because of their professional interests in their own tasks. Knowledge transfer via communication occurs even beyond the boundary of a division or company based on communities of practice (Malone 2001). This study assumes an additive property for individuals' knowledge, and the accumulated knowledge that a member receives by communicating with other members is the sum of others' knowledge.[2] The level of knowledge that each member can receive through learning-from-others depends on two factors: the level of communication adopted in the EIP and others' willingness to provide information.

Assumptions Regarding the Three Learning Processes

In general, both breadth and depth of knowledge are enhanced by all three types of learning processes. In normal situations, it is reasonable to assume that EIP members obtain broad knowledge by investing in learning so that they can expand the breadth of knowledge. On the other hand, each individual member is more likely to enhance their own knowledge in terms of its depth through experiences from working on the task than through investment for learning. Therefore, even though in practice, an EIP member may be able to enhance both breadth and depth of knowledge in all three learning processes, for the purposes of this paper, the significant difference between the two is assumed to be learning-by-investment versus learning-by-doing. We assume that the learning-by-investment process mostly affects the breadth of knowledge (i.e., the productivity of learning-by-investment is higher when the investment is focused on the breadth of knowledge than when it is focused on the depth of knowledge), whereas the learning-by-doing process primarily influences the depth of knowledge (i.e., the productivity of learning-by-doing is higher when the investment is concentrated on the depth of knowledge than when it is on the breadth of knowledge). The impact of the learning-from-others process on the breadth or depth of knowledge mainly depends on the expertise that each individual member

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[2] The "additiveness" assumption of knowledge in this paper is based on the argument that whenever knowledge is exchanged, it is assumed that it is always knowledge that is meaningful to the recipient and that recipient's knowledge is not echoed back to him or her or that the same piece of knowledge is not endlessly circulated. Although a specific piece of knowledge can be shared by more than one person, each person has their own interpretation scheme where one may understand the piece in a unique way to internalize it (Orlikowski 2002) and the same piece of knowledge may be utilized in various levels depending on the receiver's absorptive capacity (Cohen and Levinthal 1990).

possesses and that is available to transfer.[3] Active communications between members who conduct different tasks promote learning-from-others between different divisions (Dewett and Jones 2001).

It is also assumed that the balanced knowledge accumulation of each individual member in terms of both breadth and depth of knowledge is desirable for the EIP organization as well as individual members, and the individual's learning model maximizes both breadth and depth of knowledge. Although organizations can achieve the breadth of knowledge through well structured division of labor without relying on individuals' breadth of knowledge, it is still desired that each individual member enhances the breadth of knowledge such that the entire work process can be done efficiently (Campion and McClelland 1993). This is because, when individuals understand the whole process better, they proactively act to improve the overall performance.

Mathematical Model

The proposed model assumes that an individual participates in the EIP for the purpose of obtaining the maximum level of knowledge by optimizing decisions regarding learning processes. Suppose that an individual member *i* participates in an EIP. It is assumed that the individual stays at the EIP for a sufficient period of time—from time *1* when the member begins to participate in the EIP to time *T* when he or she exits the EIP—so that he or she can be called a member of the EIP. At time *0,* the member *i* is assumed to possess a certain initial level of knowledge before he or she enters the EIP. Table 1 explains the notations that are used in the mathematical model.

The problem, represented by the following mathematical model, is to maximize the net profit of knowledge acquired by member *i* over the time period from *0* to *T* with equations defining the three types of knowledge acquisition processes and the initial level of knowledge at the time of entry. The entire knowledge acquisition process generates cumulative knowledge in terms of its breadth ($B_{it} = \int_0^t b_{i\tau} d\tau$) and depth ($D_{it} = \int_0^t d_{i\tau} d\tau$), and it is mainly controlled by member *i*'s decisions (I_{bit}, I_{dit}, E_{bit}, E_{dit}, O_{bijt}, and O_{dijt}), which are associated with distinct learning processes.

$$\underset{I_{bit}, I_{dit}, E_{bit}, E_{dit}, O_{bijt}, O_{dijt}}{Maximize} \quad V \cdot B_{iT} \cdot D_{iT} \cdot e^{-\sigma \cdot T}$$

$$-\int_0^T \begin{bmatrix} c_{1b} \cdot I_{bit} + c_{1d} \cdot I_{dit} + c_{2b} \cdot E_{bit} + c_{2d} \cdot \\ E_{dit} + c_{3b} \cdot \sum_{j \in \Phi \backslash i} O_{bijt} + c_{3d} \cdot \sum_{j \in \Phi \backslash i} O_{dijt} \end{bmatrix} \cdot e^{-\rho \cdot t}\, dt \qquad (1)$$

where

$$b_{it} = \eta_b \cdot I_{bit}^{\alpha_b} \cdot \left[E_{bit} \cdot B_{it}\right]^{\beta_{bt}} \cdot \left[\theta_b \cdot \sum_{j \in \Phi \backslash i} \left(O_{bijt} \cdot w_j \cdot B_{jt}\right)\right]^{\gamma_b} \qquad (2)$$

$$d_{it} = \eta_d \cdot I_{dit}^{\alpha_d} \cdot \left[E_{dit} \cdot D_{it}\right]^{\beta_{dt}} \cdot \left[\theta_d \cdot \sum_{j \in \Phi \backslash i} \left(O_{dijt} \cdot w_j \cdot D_{jt}\right)\right]^{\gamma_d} \qquad (3)$$

$$\beta_{bt} = \frac{\mu}{1 + \delta \cdot e^{-1/\pi \cdot N_t}} \qquad (4)$$

$$\beta_{dt} = \frac{\lambda}{1 + \varepsilon \cdot e^{-\pi \cdot N_t}} \qquad (5)$$

$$B_{i0} = \underline{B}_i \qquad (6)$$

$$D_{i0} = \underline{D}_i \qquad (7)$$

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[3] It could be argued that the depth of knowledge can be enhanced by receiving knowledge from other members who belong to the same division (i.e., they conduct the same task as the knowledge recipient), and the breadth of knowledge is enhanced by learning from other members who belong to different divisions. However, the proposed model does not explicitly incorporate this feature of the learning-from-other process due to the extra degree of complexity that it would introduce in the model analysis. We leave this issue for future research.

Table 1. Model Notation

Variables	Description
T	Time when member *i* exits the EIP
I_{bit} , I_{dit}	Rate of investment made by member *i* to enhance the breadth (*b*) (depth (*d*)) of knowledge at time *t (learning-by-investment)*
E_{bit} , E_{dit}	Rate of effort made by member *i* to enhance the breadth (*b*) (depth (*d*)) of accumulated knowledge by conducting tasks at time *t (learning-by-doing)*
O_{bijt} , O_{dijt}	Rate of effort made by member *i* to receive the breadth (*b*) (depth (*d*)) of knowledge from member *j* at time *t (learning-from-others)*
b_{it}	Rate of the breadth of knowledge acquired by member *i* at time *t*
d_{it}	Rate of the depth of knowledge acquired by member *i* at time *t*
B_{it}	Cumulative level of the breadth of knowledge acquired by member *i* at time *t*
D_{it}	Cumulative level of the depth of knowledge acquired by member *i* at time *t*
β_{bt} , β_{dt}	Exponent of the learning-by-doing process to produce the breadth (*b*) (depth (*d*)) of accumulated knowledge
w_j	Scale of willingness to transfer member *j*'s knowledge
Φ	Group of all members in the EIP
N_t	Total number of members participating in the EIP at time *t*
$\underline{B}_i$	Initial breadth of knowledge possessed by member *i* at the time of entry
$\underline{D}_i$	Initial depth of knowledge possessed by member *i* at the time of entry
V	Salvage value of knowledge
ρ	Discount rate of cost
σ	Discount rate of revenue
c_{1b} , c_{1d}	Unit cost of investment in the breadth (*b*) (depth (*d*) of knowledge
c_{2b} , c_{2d}	Unit cost of effort to enhance the breadth (*b*) (depth (*d*)) of knowledge by conducting tasks
c_{3b} , c_{3d}	Unit cost of effort to receive the breadth (*b*) (depth (*d*)) of knowledge from other members
η_b , η_d , θ_b θ_d , α_b , α_d , γ_b , γ_d , λ , μ , δ , ε , π	Constant values

The objective function (Equation (1)) demonstrates that the purpose of the proposed model is to maximize the net profit of an EIP member *i*, which is based on the revenue from accumulated knowledge at the time of exit and costs of knowledge acquisition from three learning processes during the member's participation in the EIP. The revenue indicates the value of total knowledge, which is acquired by the member *i*. It is represented as the product of the total knowledge and the unit monetary value (V). The total knowledge level is denoted as the product of the breadth ($B_{iT} = \int_0^T b_{it} dt$) and the depth ($D_{it} = \int_0^T d_{it} dt$) of knowledge, and includes the initial level of knowledge at $t = 0$ as well as the level that the member *i* acquires during his or her participation in the EIP between times $t = 1$ and $t = T$.

The objective function also contains three distinct cost items, and each cost item is associated with decision variables that are related to each of three learning processes. The first cost item ($c_{1b} \cdot I_{bit}$ and $c_{1d} \cdot I_{dit}$), which indicates the cost incurred by member *i*'s investment decision regarding the learning-by-investment process, is composed of the cost for the breadth of knowledge (I_{bit}) and the cost for the depth of knowledge (I_{dit}), while the member participates in the EIP, from time *1* to time *T*. The second cost item ($c_{2b} \cdot E_{bit}$ and $c_{2d} \cdot E_{dit}$) represents the cost that the member *i* pays for acquiring knowledge from the learning - by-doing process. Finally, the total cost in the objective function includes the costs incurred when member *i* intends to obtain knowledge from others ($c_{3b} \cdot \sum_{j \in \Phi \backslash i} O_{bijt}$ and $c_{3d} \cdot \sum_{j \in \Phi \backslash i} O_{dijt}$). Just like the cost of the learning-by-investment process, costs for learning-by-doing and learning-from-others are also defined as two different pieces depending on whether the associated decision is to acquire the breadth of knowledge or the depth of knowledge (E_{bit} and O_{bijt} for the breadth of knowledge, and E_{dit} and O_{dijt} for the depth of knowledge). All of these costs are assumed to be zero before $t = 1$, when the decisions on learning processes really begin in the EIP.

In the objective function, both the revenue and the costs are discounted so that the output (input as the cost) from the distant past is less valued than recent outputs from the point of time *0*. The discount rates (σ for the revenue and ρ for the costs) connote different meanings depending on whether it is applied to the revenue or the cost. The discount rate of the cost indicates the depreciation of the cost that is spent for acquiring knowledge over time (for example, interest rate). On the other hand, the discount rate of the revenue in the objective function (1) implies an obsolescence of knowledge. According to previous research (Argote et al. 1990; Darr et al. 1995; Epple et al. 1996; Pakes and Schankerman 1979), knowledge becomes obsolete rapidly compared with traditional capital goods. For example, the annual decay rate of usefulness of technical knowledge is more than 10 percent in the post-war period (Bosworth 1978). In general, the discount rate of the cost is not higher than the discount rate of the revenue from knowledge ($\sigma \geq \rho$).

The equations indicate the basic input-output relationship between the rate of knowledge production and the rate of learning inputs (investment, effort, accumulated knowledge, and others' knowledge). They are formulated in such a way that, by integration over a certain period of time ($B_{it} = \int_0^t b_{i\tau} d\tau$ and $D_{it} = \int_0^t d_{i\tau} d\tau$), one can construct the production function of knowledge creation. A general Cobb-Douglas production function is applied here with the assumption that accumulated knowledge is produced as the total output while the three learning processes comprise the total inputs (Lynch et al. 1990).[4] The Cobb-Douglas function has been frequently applied to

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[4] The proposed learning process may be seen as a special type of production system, which generates knowledge as an output. The knowledge acquisition process can incorporate the learning curve effect, which appears in many ordinary production systems. This 'learning curve effect in the learning process' implies that the individual can acquire new knowledge faster, as he or she accumulates more knowledge. The learning curve effect can be expressed as a certain function and added to the first two equations in the proposed model. However, the current model does not incorporate this effect, because of the extra degree of complexity that it would introduce. We leave this issue for future research.

represent a general input-output relationship in various areas including the learning process (Dorroh et al. 1986, 1994; Gulledge et al. 1984).

Three separate knowledge acquisition processes and their outcomes are represented in terms of two attributes, breadth (Equation (2)) and depth (Equation (3)). First, knowledge is acquired by individual investment of two types—one to deepen knowledge (I_{dit}) and the other to broaden knowledge (I_{bit}).

Second, the acquired knowledge depends on the formerly accumulated knowledge (B_{it} and D_{it}) and the decision to acquire knowledge through learning-by-doing (E_{bit} and E_{dit}). In this process effect, the individual obtains knowledge while performing tasks of the EIP.

β_{bt} and β_{dt} represent the exponents of the learning-by-doing process. Equations (4) and (5) show how to determine the exponents of the learning-by-doing process at time *t*. Division of labor mainly affects these exponents. It is assumed that the division of labor intensifies as the number of members in the EIP (N_t) increases. These two equations describe a relationship between the degree of division of labor and the knowledge level obtained. In order to specify the range from the worst to the best situation of knowledge acquisition depending on the division of labor, logistic functions are used in Equations (4) and (5) (De Liso et al. 2001). In Equation (4) with the positive value of π, the exponent for the breadth of knowledge (β_{bt}) decreases as the number of members (N_t) increases, ranging from μ when $N_t \rightarrow 0$ 1 to $\frac{\mu}{1+\delta}$ when $N_t \rightarrow \infty$, where $0 \leq \delta \leq 1$. In Equation (5), the exponent for the depth of knowledge (β_{dt}) increases as the number of members increases, ranging from $\frac{\lambda}{1+\varepsilon}$ when $N_t \rightarrow 0$ to λ when $N_t \rightarrow \infty$, where $0 \leq \varepsilon \leq 1$.

The third component of Equations (2) and (3) represents the learning-from-others effect. This type of learning process is dependent on the other members' accumulated knowledge so far (B_{jt} and D_{jt}, where $j \in \Phi \setminus i$) and their willingness to transfer it to the member *i* (w_j). Member *i*'s decision to receive knowledge from another member *j* (O_{bijt} and O_{dijt}) determines the ultimate amount of knowledge acquired via this learning process. The value of θ_b represents the level of technology installed in the EIP to support communication among members who conduct distinct tasks. This type of technology enhances the breadth of knowledge. θ_d indicates support of communication among members within the same function or division, and this type of communication improves the depth of specialized knowledge.

Equations (2) and (3) represent the unique property of each learning process. Depending on the existence of additional inputs and the source of additional inputs, the proposed model formulates the special characteristics of different learning processes as different mathematical formulas. The main difference of the learning-by-investment from the other two processes is that it contains a single input, which is investment as a decision variable. On the other hand, the other learning processes have additional inputs other than the decisions regarding efforts (i.e., accumulated knowledge for the learning-by-doing process and transferred knowledge for the learning-from-others process). Although both learning-by-doing and learning-from-others processes have additional inputs other than the decision variables, their additional inputs have different sources. For the learning-by-doing process, the additional input is the knowledge that was accumulated in the past, and is internal to the entire learning process. On the other hand, the additional input for the learning-from-others process is knowledge that is transferred from other members, and it comes from outside of the member's learning process. The mathematical expressions in the proposed model reflect the unique property of each learning process.

The last two equations indicate the initial level of knowledge, in terms of breadth (Equation (6)) and depth (Equation (7)), which the individual member possesses at the time of entrance into the EIP. Since any participant of the EIP is restricted to be a formal member who is ready to start on the task at time *1*, the initial level of knowledge is assumed to be greater than zero ($B_{i0} = \underline{B}_i > 0$ and $D_{io} = \underline{D}_i > 0$).

Numerical Examples and Analyses

The numerical examples are analyzed with three basic objectives. First, we examine the impact of several parameters on the profit: the numerical analysis is conducted to find favorable conditions in which the individual EIP member can improve profit from the learning processes.

Second, based on the favorable conditions detected from the first analysis, the analysis attempts to identify the dynamic nature of decisions that the individual member should make in order to achieve the optimal profit from learning during his or her participation in the EIP. In the second part of analysis, two outputs become the main points of interest: decisions on three learning processes (I_{bit}, I_{dit}, E_{bit}, E_{dit}, O_{bijt}, and O_{dijt}) and accumulated knowledge (B_{it} and D_{it}). Investment and effort for learning processes are the major decisions in the proposed model, and they represent the individual's plan for resource consumption required during his or her participation in the EIP. Therefore, it is critical to determine the nature of the optimal decision plan over a certain time period and its changes due to the alteration of environmental factors as well as changes in the magnitude of knowledge accumulated over time, since the impact of optimal decisions on the profit depends on various parameters. The analyses focus on both timing and magnitude of optimal investment decisions and accumulated knowledge.

Finally, the analysis of the numerical examples investigates the effect when a specific learning process is more productive than others. Here, the productivity is defined as the rate of knowledge accumulation and expressed as the exponents in the mathematical model (learning-by-investment α, learning-by-doing β, and learning-from-others γ). This analysis examines the existence of interdependency among three learning processes in terms of their productivity.

The simplified mathematical model for the numerical analyses is represented below.

$$\underset{I_{bt}, I_{dt}, E_{bt}, E_{dt}, O_{bt}, O_{dt}}{Minimize} -V \cdot B_T \cdot D_T \cdot e^{-\sigma \cdot T} + \int_0^T \left[I_{bt} + I_{dt} + E_{bt} + E_{dt} + O_{b\tau} + O_{d\tau} \right] \cdot e^{-\rho \cdot t}\, dt \tag{8}$$

where

$$b_t = \eta_b \cdot I_{bt}^{\alpha_b} \cdot \left[E_{bt} \cdot B_t \right]^{\beta_{bt}} \cdot \left[\theta_b \cdot O_{bt} \cdot Z_{bt} \right]^{\gamma_b} \tag{9}$$

$$d_t = \eta_d \cdot I_{dt}^{\alpha_d} \cdot \left[E_{dt} \cdot D_t \right]^{\beta_{dt}} \cdot \left[\theta_d \cdot O_{dt} \cdot Z_{dt} \right]^{\gamma_d} \tag{10}$$

$$\beta_{bt} = \frac{\mu}{1 + \delta \cdot e^{-1 / \pi \cdot N_t}} \tag{11}$$

$$\beta_{dt} = \frac{\lambda}{1 + \varepsilon \cdot e^{-\pi \cdot N_t}} \tag{12}$$

$$B_0 = \underline{B} \tag{13}$$

$$D_0 = \underline{D} \tag{14}$$

In Equations (9) and (10), the rate of effort made by member *i* to receive knowledge from member *j* (O_{bijt} and O_{dijt}) is simplified into the rate of total efforts made by member *i* to obtain the knowledge from all other members in an EIP (O_{bt} and O_{dt}). Z_{bt} (Z_{dt}) is defined as the total amount of the breadth (b) (depth (d)) of knowledge transferred from other members ($Z_{bt} = \sum_{j \in \Phi \setminus i} \left(w_j \cdot B_{jt} \right)$ and $Z_{dt} = \sum_{j \in \Phi \setminus i} \left(w_j \cdot D_{jt} \right)$), and it is expressed as an increasing function of time in the numerical examples. In addition, the costs to enhance every individual learning process are assumed to be identical ($c_{1b} = c_{1d} = c_{2b} = c_{2d} = c_{3b} = c_{3d} =$ *1*), because the main goal of numerical examples is to observe the relative patterns of optimal decisions and resultant knowledge levels rather than the absolute value of the optimal profit.

Optimal solutions can be obtained by applying optimal control theory. Although it is not feasible to attain closed-form solutions of I_{bt}, I_{dt}, E_{bt}, E_{dt}, O_{bt}, O_{dt}, B_t, and D_t in this problem, with the given specified parameters, the numerical optimal solution can be provided by applying the shooting procedure. Detailed steps of the shooting procedure are described in Appendix A. Several moderate sized numerical examples are considered and analyzed. In each numerical example, a parameter is set to a different value. For comparison among different cases, the base case parameters are arbitrarily chosen (see Table 2). This numerical example assumes that the size of EIP grows over time, where the number of EIP members increases with time $(N_t = 15 \bullet t + 10)$. Due to this increase, the amount of knowledge transferred from others is also an increasing function of time $(Z_{bt} = Z_{dt} = 3 \bullet t = 10)$. This assumption indicates that the EIP grows over time but it does not account for any mature EIPs, which stop its growth or go into decline.

The base case parameters indicate the general case that the difference in the productivity of learning processes exists depending on the attribute of knowledge. Recall that we assume $\alpha_b > \alpha_d$ for the learning-by-investment process and $\beta_b < \beta_d$ for the learning-by-doing process. Meanwhile, the total return to scales of the overall learning processes are identical for both breadth and depth of knowledge, when the division of labor effect is ignored ($\alpha_b + \mu + \gamma_b = \alpha_d + \lambda + \gamma_d = 0.3$).

Impacts of Environmental Factors on Profit

In the research model, we discuss seven environmental factors influencing the knowledge accumulation and the profit from the accumulated knowledge relative to costs for knowledge acquisition. These factors are the discount rate of cost (*rho (ρ)*), discount rate of revenue (*sigma (σ)*), salvage value of knowledge (*V*), the initial level of knowledge (*B(0)* and *D(0)*), the number of EIP members (*N(t)*), other's knowledge (*Z(t)*), and the exponent of the learning-by-doing process (*mu* (μ) and *lambda (λ)*). In the rest of this section, we first present the results and then develop a set of testable propositions based on results from the mathematical model and some prior literature (in a similar vein to the work of Carley and Lin 1997).

Figure 2 shows how the optimal profit changes due to the different values of seven environment factors. Table 3 shows the specific values of each parameter used in the examples. According to Figure 2, the optimal profit increases as the discount rate of cost, the salvage value of knowledge, the initial level of knowledge, and the exponent of the learning-by-doing process increase. The same effect is observed when others' knowledge and the number of EIP members increase rapidly over time. On the other hand, the optimal profit is lower when the discount rate of revenue is higher.

As expected, the analysis of profit reveals that the lower discount rate of revenue and the higher salvage value of knowledge would have a positive impact on the revenue. Higher discount rate of cost results in greater profit due to the reduced cost. The increased profits under the conditions of larger initial knowledge level, faster increasing others' knowledge, and higher exponent of the learning-by-doing process imply that the EIP member is able to achieve better performance in a more favorable situation of the knowledge acquisition process.

Figure 2 indicates that the impact of the number of EIP members on the optimal profit is different from our expectations. It is expected that the intensified division of labor due to the rapid increase in number of EIP members has both positive and negative impacts on the ultimate knowledge level that are of similar order (i.e., it has a positive impact on the depth of knowledge, but it negatively affects the breadth of knowledge). Accordingly, the impact of the number of EIP members on the total profit is expected to be minimal due to both positive and negative impacts of the number of

Table 2. Parameters Used in a Base Case

$T = 30$	$V = 1.0$
$\rho = 0.03$	$\sigma = 0.03$
$\alpha_b = 0.15$ $\alpha_d = 0.05$	$\gamma_b = \gamma_d = 0.08$
$\mu = 0.07$ $\lambda = 0.17$	$\delta = \varepsilon = 0.7$
$\pi = 0.005$	$\eta_b = \eta_d = 0.7$
$\theta_b = \theta_d = 0.7$	$\underline{B} = \underline{D} = 15$
$N_t = 15 \cdot t + 10$	
$Z_{bt} = Z_{dt} = 3 \bullet t + 10$	

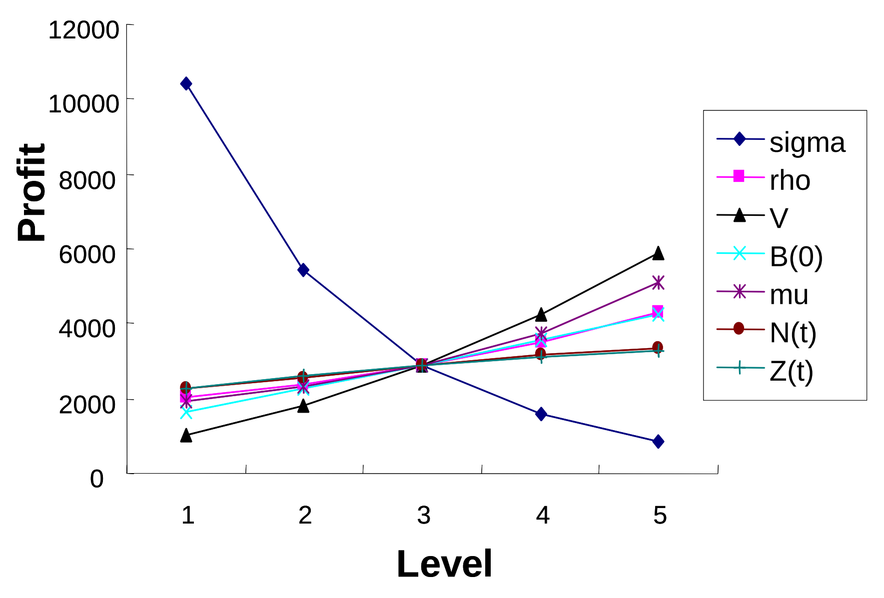


Figure 2. Impacts of Parameters on Total Profit

Table 3. Levels of Parameters Used in the Numerical Example

Level	1	2	3	4	5
sigma	0.01	0.02	0.03	0.04	0.05
rho	0.01	0.02	0.03	0.04	0.05
V	0.6	0.8	1.0	1.2	1.4
B(0) (D(0))	5	10	15	20	25
N(t)	= 5 * t + 10	= 10 * t + 10	= 15 * t + 10	= 20 * t + 10	= 25 * t + 10
mu lambda	0.05 0.15	0.06 0.16	0.07 0.17	0.08 0.18	0.09 0.19
$Z_b(t)$ ($Z_d(t)$)	= 1 * t + 10	= 2 * t + 10	= 3 * t + 10	= 4 * t + 10	= 5 * t + 10

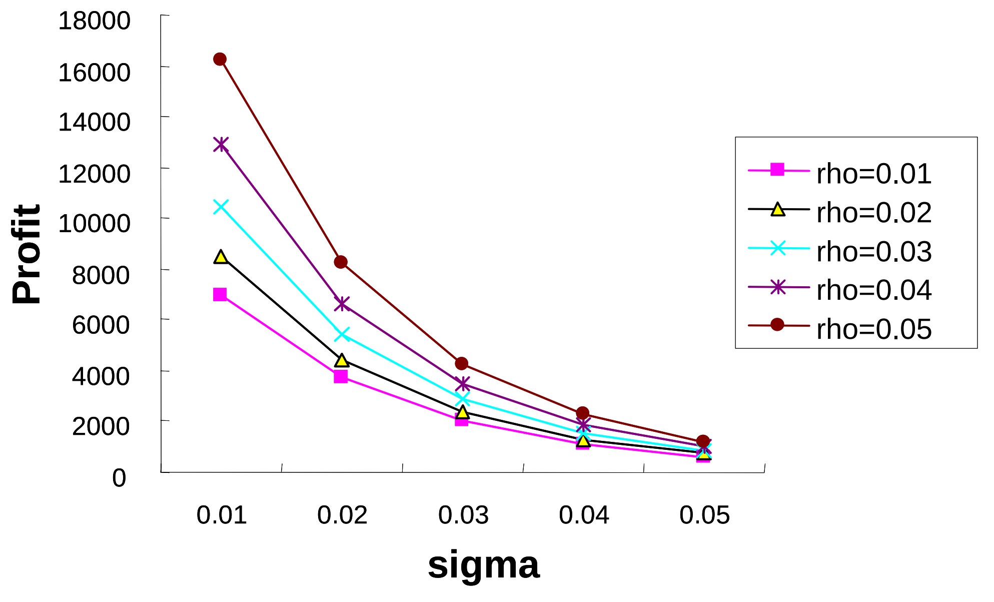


Figure 3. Interaction Effect on the Total Profit (Discount Rate of Cost Versus Discount Rate of Revenue)

EIP members on the accumulated knowledge. However, the outcome shows the positive impact of the number of EIP members on the optimal profit. The reason for this result is that the current example sets the exponent for the depth of knowledge much higher than for the breadth of knowledge in the learning-by-doing process ($\mu = 0.07 < \lambda = 0.17$). The number of EIP members solely affects the exponents of the learning-by-doing process (see Equations (11) and (12)). In this situation, the positive impact of the number of EIP members on the depth of knowledge expands with the given high exponent ($\lambda = 0.17$), while its negative impact on the breadth of knowledge becomes negligible due to the low exponent ($\mu = 0.07$). The result shows only the positive impact of the number of EIP members on profit.

Further analyses are conducted on the optimal profit in order to detect the possible interaction effect among different parameters. In this analysis, however, no significant interaction between different parameters is found. For example, Figure 3 shows the interaction effect of two environmental factors—the discount rate of revenue (*sigma*) and the discount rate of cost (*rho*)—on the optimal profit. This graph implies that the EIP member can achieve greater performance in the learning process when the decay rate of knowledge is low (low *sigma*) and the capital investment and expenses can be made to acquire knowledge at the high discount rate (high *rho*).

Based on the above results, we can specify the following proposition for possible further investigation. (To save space, we have collapsed multiple propositions into a single proposition.)

Proposition 1. *Factors such as lower discount rate of revenue, higher discount rate of cost, higher salvage value of knowledge, larger initial knowledge level, faster increase in others' knowledge, and higher productivity of the learning-by-doing process individually lead to a positive impact on profit from accumulated knowledge.*

Impact of Environmental Factors on Learning Decisions

The analyses in this part of the numerical example focus on how each environmental factor affects the dynamic natures of optimal investment and effort decisions and the resultant accumulated knowledge over a time period. Table 4 summarizes the results relating to impact of each parameter on learning decisions and leads to Propositions 2 through 8.

Figure 4 shows a sample graph of analysis outcomes described in Table 4. This result indicates the impact of discount rate of cost on investment for breadth of knowledge. When the line associated with the high discount rate of cost (rho = 0.05) is compared with one with the low discount rate of cost (rho = 0.01), the distance between them increases over time. In particular, the distance between them at time 30 is much larger than at time 0. The behavior can be interpreted as follows: Rising discount rate of cost leads to proportionately more investment being made during later periods. This has interesting implications. When the discount rate of costs is high, it is advisable for an EIP member to delay a relatively large portion of the total investment until the later point of the given time, in order to obtain the optimal level of accumulated knowledge. On the other hand, when the discount rate of costs is low, the total investment should be evenly distributed over time.

Impact of Discount Rate of Cost

The main motivation for analyzing the discount rate of cost is that it allows differentiation between situations of knowledge creation where different types of resources are consumed. In general, when the resources spent to create knowledge involve liquid assets such as currency, the discount rate is high. However, if most of the required resources are fixed assets, the discount rate is considered relatively low.

Table 4. Optimal Decisions Regarding Investment and Effort for Learning at Given Conditions

Conditions	Size of Optimal Decisions on Investment and Effort for Breadth of Knowledge	Time of Optimal Decisions for Breadth of Knowledge	Size of Optimal Decisions for Depth of Knowledge	Time of Optimal Decisions for Depth of Knowledge
High discount rate of costs	Large	Delayed	Large	Delayed
Low discount rate of costs	Small	Evenly distributed	Small	Evenly distributed
High discount rate of revenue	Small	Evenly	Small	Evenly
Low discount rate of revenue	Large	Delayed	Large	Delayed
High salvage value of knowledge	Large	Delayed	Large	Delayed
Low salvage value of knowledge	Small	Evenly	Small	Evenly
High level of initial knowledge	Large	Delayed	Large	Delayed
Low level of initial knowledge	Small	Evenly	Small	Evenly
Fast increase in number of EIP members	Large	Delayed	Large	Delayed
Slow increase in number of EIP members	Small	Evenly	Small	Evenly
High productivity in learning-by-doing	Large	Delayed	Large	Delayed
Low productivity in learning-by-doing	Small	Evenly	Small	Evenly
Fast increase in others' knowledge	Large	Delayed	Large	Delayed
Slow increase in others' knowledge	Small	Evenly	Small	Evenly

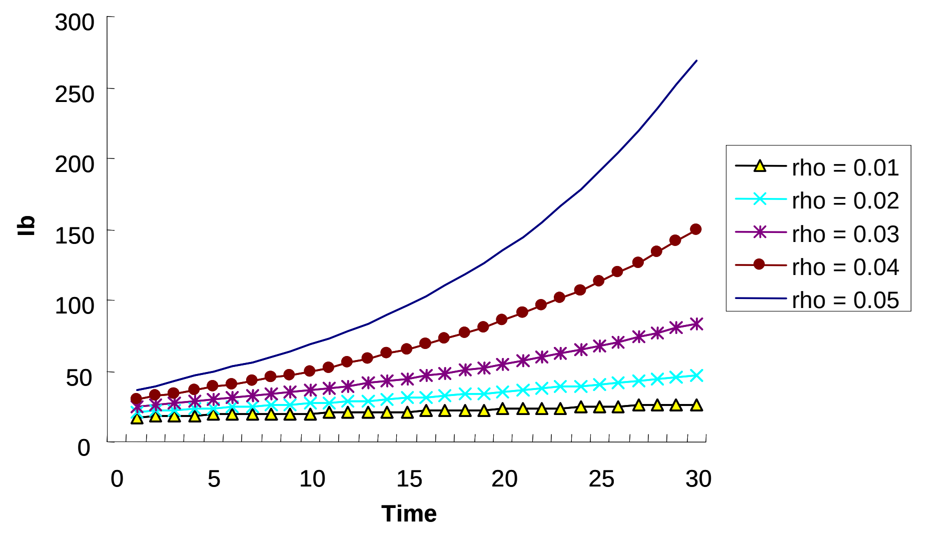


Figure 4. Impact of Discount Rate of Cost on Investment for Breadth of Knowledge

Table 4 shows, across the three learning processes, the tendency of investment postponement to obtain the optimal level of knowledge in terms of its breadth and depth when the discount rate is high. As the discount rate of cost increases, the optimal decision is to invest more resources later. On the other hand, when the discount rate is moderate or low, relatively larger investment is required at an early stage.

Discounting makes the early investment and efforts expensive over the time in which the individual member remains in the EIP. When the discount rate is relatively high, the individual member is motivated to postpone his or her investment and efforts until a later point of the given time period (Dorroh et al. 1986). The high discount rate can be interpreted as a high opportunity cost for individual members in the EIP. Proposition 2 proposes this relationship between the discount rate and timing of the investment decision.

Proposition 2. *Higher discount rate of investment and effort leads to an optimal behavior characterized by increased and delayed investment in learning.*

Impact of Discount Rate of Revenue

The revenue in the objective function of the proposed model represents the product of the salvage value and total knowledge. The knowledge acquired through the learning processes does not conserve its value indefinitely, and instead it depreciates over time (Epple et al. 1996). The obsolescence of knowledge is due to the difficulty in maintaining the ability to obtain its benefit and update of new innovation, which partly or entirely displace the past knowledge (Pakes and Schankerman 1979). Previous studies observed the relatively high decay rate of knowledge compared with the traditional capital goods (Argote et al. 1990; Bosworth 1978; Darr et al. 1995). The exact decay rate of knowledge depends on the durability of knowledge, which may be closely related with the nature of knowledge. For example, the rate of obsolescence of research knowledge in physics and chemistry is much higher than ones in history and English (McDowell 1982). In general, the discount rate of revenue (σ) is higher than the discount rate of cost (ρ), which is ordinary capital spending in most cases. However, the exact discount rate of revenue must depend on the nature of the specialized knowledge, which the individual member intends to acquire from the EIP.

Table 4 indicates that when the revenue of knowledge is higher due to lower discount rate, a larger investment is required to obtain the optimal performance than in the case of lower revenue from accumulated knowledge (higher discount rate) across the three learning processes. This result also indicates that the member ought to distribute investment evenly over time in the case of less durable knowledge such as high technology knowledge (high discount rate of revenue). On the other hand, if the member intends to acquire durable knowledge such as low technology industry knowledge during participation in the EIP (low discount rate of revenue), the postponement of investment is the preferable plan. Proposition 3 explains the relationship between the discount rate of revenue and the optimal investment and effort decision plan.

Proposition 3. *Under conditions of low discount rate of accumulated knowledge, increased and delayed learning investment and efforts contribute to higher cumulative knowledge.*

Impact of Salvage Value of Knowledge

Table 4 indicates that when knowledge appreciates at a high price (high salvage value), the individual member of an EIP is motivated to increase total resource consumption for knowledge creation over time so that the member can obtain a much greater level of knowledge. If the salvage value of knowledge is low, however, it is preferable to distribute the whole effort evenly over the given period.

The real value of knowledge can be determined by various factors, including industry structure, related work, knowledge users, and the nature of knowledge. In general, knowledge is greatly appreciated in high technology industries, where it is considered to be one of the critical assets for operation or competitive advantage. When the value of knowledge is high enough to cover the cost to create it, the member is motivated to spend more resources to generate knowledge. It is preferable to make an intensive consumption in the later stage of learning processes, when its cost is cheaper due to the discount rate of cost. Otherwise, a small amount of evenly distributed resource allocation over time is considered better. Hence, Proposition 4 is proposed.

Proposition 4. *Higher salvage value of knowledge leads to optimal behavior characterized by increased and delayed investment and efforts in learning*.

Impact of Initial Knowledge

According to Table 4, the member of the EIP can optimize profit by making a larger investment and effort to learn from other members across the three learning processes when the member begins with a higher level of initial knowledge. Compared with the initial effort required in the optimal plan, greater effort is required as the plan starts with a higher level of initial knowledge.

The high level of initial knowledge provides a favorable condition and enables the member to optimize the profit by making a large effort, and larger effort is still required as time goes on. On the other hand, the level of initial knowledge affects the learning process only at the time of entry (at $t = 1$) contrary to other factors that have continuous effects over the time.

In general, a sufficient stock of knowledge before an individual member enters the EIP would deliver considerable benefits for future knowledge creation because the initial stock of knowledge becomes an input in the learning-by-doing process. A large amount of initial knowledge is also expected to reduce the burden of later investment and efforts to obtain the optimal level of knowledge at the end of the time period. Proposition 5 follows this line of reasoning.

Proposition 5. *Under conditions of high initial knowledge level, increased and delayed learning investment and efforts lead to higher cumulative knowledge*.

Impact of Number of EIP Members

The result of the numerical examples in Table 4 indicates that when the number of EIP members increases at a high rate, greater effort is required to obtain the optimal level of knowledge in terms of both its breadth and depth across the three learning process. The resulting accumulated knowledge becomes higher when the number of EIP members increases at a high rate rather than at a low rate. The result for the breadth of knowledge is different from the expectation, while the outcome matches the expectation for the depth of knowledge.

We assume that the increased size of the EIP encourages each member to improve the depth of knowledge due to the intensified division of labor. On the other hand, specialized task assignment under a high level of division of labor reduces the opportunity to improve breadth of knowledge for each member. This relationship between the learning process and division of labor is reflected in the proposed model (Equations (4) and (5)). However, the numerical examples show a different result from the expectation. Based on the given model design, the result was expected to show that greater breadth of knowledge results from greater effort requirement when the number of EIP members increases at a low rate than when it increases at a high rate.

In order to examine this discrepancy between actual outcome and theoretical expectation, further analysis is conducted on a different numerical example. In the alternative example, the parameter specification is different from the base case of the previous example—i.e., base exponents of processes are identical for both breadth and depth of knowledge (now, α_b = 0.1 α_d = 0.1, μ = 0.12, and λ = 0.12). In the result of this analysis, the optimal learning effort shows a different pattern from the previous example. When the number of EIP members increases at a low rate, less effort is required at the early time. On the other hand, the optimal plan requires greater effort than when the number of EIP members increases at the high rate after it passes a certain time. This outcome matches the theoretical expectation. The resultant accumulated knowledge indicates the same result—i.e., a higher level of knowledge (breadth) results when the number of EIP members increases at the low rate. The result for the depth of knowledge is same as the previous example as it was expected based on the model specification. More resources are required to generate greater depth of knowledge when the number of EIP members increases at the higher rate.

We conjecture that the outcome of the prior example, which contradicts the theoretical expectation, results from the particular parameter specification. The base numerical example specifies a much lower base exponent of the learning-by-doing process for the breadth of knowledge (μ = 0.07) than for the depth of knowledge (λ = 0.17). Since the number of EIP members affects only the exponent of the learning-by-doing process, its impact is highly sensitive to this base exponent (μ and λ). Although the increased number of EIP members has a negative impact on the breath of knowledge, its impact is minimal due to the low value of base exponent (μ). This effect manifests when it is compared with the case where the base exponents of all processes are identical (α_b = α_d and μ = λ). In this case, greater effort is required and results in greater breadth of knowledge as the number of EIP members increases at the low rate, while the higher level of efforts is spent to generate more knowledge in its depth as the number of members increases at the high rate. Moreover, the outcome of total profits shows that the total profit does not change as the number of EIP members increases at a higher rate. By implication, when the base exponents of the learning-by-investment and learning-by-doing processes are identical for breadth and depth of knowledge, the impact of the number of EIP members contains a certain trade-off between breath and depth. These results are summarized as Propositions 6 (a) and 6 (b).

Proposition 6(a). *In general, the number of members positively affects depth of knowledge but negatively affects breadth of knowledge.*

Proposition 6(b). *When the learning-by-investment process is more productive for the breadth of knowledge than for its depth and simultaneously the learning-by-doing process* *is more productive for the depth of knowledge than for its breadth, the number of members positively affects both breadth and depth of knowledge.*

Impact of Productivity of Learning-by-Doing Process

The impact of the productivity of the learning-by-doing process on patterns of accumulated knowledge is investigated by making changes in the value of μ and λ, in the function of β for the learning-by-doing process, which lead to changes in the return to scale of the knowledge acquisition process. Here, the exponents of the learning-by-doing process (β_b and β_d) are the precursors of productivity.

Table 4 indicates that when the value of the exponent increases, more investment is required to obtain optimal knowledge. Early investment required for the optimal plan increases significantly as the value of the exponent increases, and the amount of investment required at the end of the time period increases even further. This result implies that the postponement of investment is optimal when the learning-by-doing process is highly productive. On the other hand, if the productivity of the learning-by-doing process is low, a relatively small amount of additional investment is required later compared with the initial investment to achieve optimal performance. Accordingly, knowledge accumulated in the highly productive knowledge production system becomes much greater than in the system with low productivity.

In conclusion, when an EIP has a productive process for accumulating knowledge, the individual members are motivated to make intensive investment and efforts during their participation, and they are eventually able to obtain a large amount of knowledge (Gulledge et al. 1984). The relationship between productivity and accumulated knowledge is in Proposition 7.

Proposition 7. *Higher productivity of the learning-by-doing process leads to an optimal behavior characterized by increased and delayed investment and efforts in learning.*

Impact of Others' Knowledge

Knowledge transferred from other EIP members is a major input of the learning-from-others process. In this study, the total knowledge transferred from other members is defined as an increasing function of time, because it increases over time as the number of members augments and their stay is extended. The amount of knowledge transferred from other members is determined by two factors in the proposed model: support of an EIP that is made to enhance either communication among members between distinct divisions to improve the breadth of knowledge (θ_b in Equation (2)) or communication among members within the same division to improve the depth of knowledge (θ_d in Equation (3)) and other members' willingness to transfer their own knowledge to him or her (w_j in Equations (2) and (3)).

Table 4 shows that greater effort is required to obtain the optimal level of knowledge when the rate of increasing others' knowledge is high. The difference in the amount of effort required between high and low rates of growth in others' knowledge are small at the initial stage, but the amount gradually increases over time and becomes a significant disparity. When an EIP provides intense support for active communication to members and thereby increases their willingness to transfer knowledge to others, each member is motivated to make rigorous investments in knowledge creation and eventually obtains a large amount of knowledge. This effect is described in Proposition 8.

Proposition 8. *Higher knowledge receipt from other members leads to an optimal behavior characterized by increased and delayed investment and efforts in learning*.

Impact of Productivity of a Learning Process on Other Processes

The current study investigates the effect of a more productive learning process on the other processes in terms of their interaction. As discussed earlier, in the proposed model, the productivity of a learning process is defined as the return to scale of the process. For the example, the exponent (or base exponent in the case of the learning-by-doing process) is set higher for one learning process than the remaining two processes, and the analysis examines the resulting optimal decisions on three learning process and the effect of accumulated knowledge.

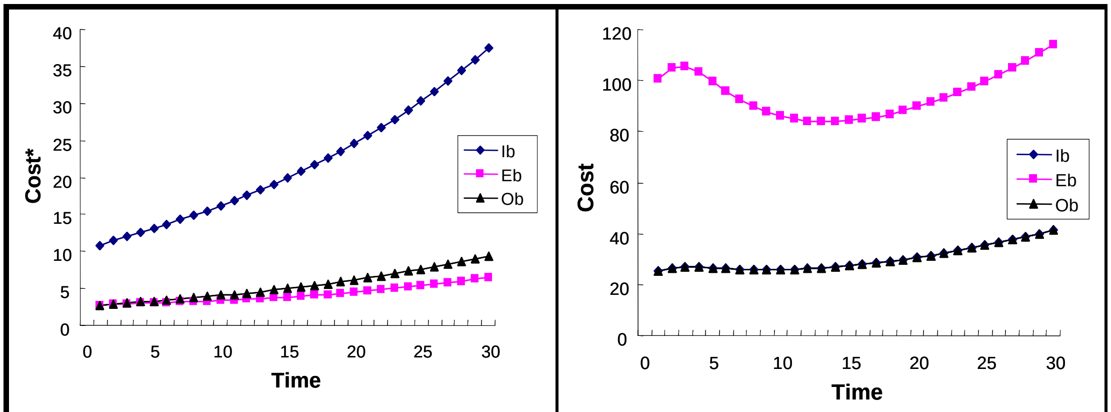
Figure 5 (a) shows the result when the learning-by-investment process is more productive than the other process (α_b = α_d = 0.2. $\mu = \lambda$ = 0.05, γ_b = γ_d = 0.05). This figure indicates the clear tendency that significantly more resources should be committed to the learning-by-investment process than other processes in order to obtain the maximum performance in terms of breadth of knowledge. This result is intuitive in that more resources must be allocated to a more productive process than the others. The same result is observed when the other learning processes are more productive. Figures 5(b) and 5(c) show the results when the learning-by-doing process is more productive than others (α_b = α_d = 0.05. μ = λ = 0.2, γ_b = γ_d = 0.05), and the result of the case that the learning-from-others is more productive than other processes is illustrated in Figure 5(d) (α_b = α_d = 0.05. $\mu = \lambda$ = 0.05, γ_b = γ_d = 0.2). These results imply that, in general, more resources should be assigned to a more productive learning process in order to obtain the optimal performance.

Meanwhile, Figure 5(b) shows a unique nature of the learning-by-doing process for the breadth of knowledge. In the case that the learning-by-investment (Figure 5(a)) or learning-from-others process (Figure 5(d)) is more productive than the other two learning process, the optimal investment increases over time. Even when the learning-by-doing process is more productive for the depth of knowledge than other two processes, the optimal investment also increases over time (Figure 5(c)). On the other hand, when the learning-by-doing process is more productive for the breadth of knowledge than other two processes, the optimal decision is characterized by a large amount of effort at the beginning of the time period and the optimal effort decreases in the middle of the whole time period and increases again toward the end.

The phenomenon can be explained as follows: The main input of the learning-by-doing process is the knowledge that had been accumulated at the previous time, as it is described in Equations (9) and (10). Therefore, when the learning-by-doing process is highly productive, the intensive effort made at the early time help obtain a large amount of accumulated knowledge at a later time. This explains why optimal effort distribution shows an intensive early investment when the learning-by-doing process is more productive than other two learning processes. With passage of time, the increased number of EIP members decreases the productivity of the learning-by-doing process for the breadth of knowledge (see Proposition 6(a)). Since the example assumes that the number of EIP members increases over time (see parameter setting for the example in Table 2), the passage of time makes the investment in the learning-by-doing process for the knowledge breadth less productive. Accordingly, the optimal effort made for the breadth of knowledge decreases in the middle of the time period. However, toward the end of the given time period, the increased effort made for the learning-by-doing process is optimal due to the growing impact of the discount rate of cost. This unique nature of the learning-by-doing process for the breadth of knowledge appears when the productivity of this learning process is greater than that of the other two processes. This result is described in Proposition 9.

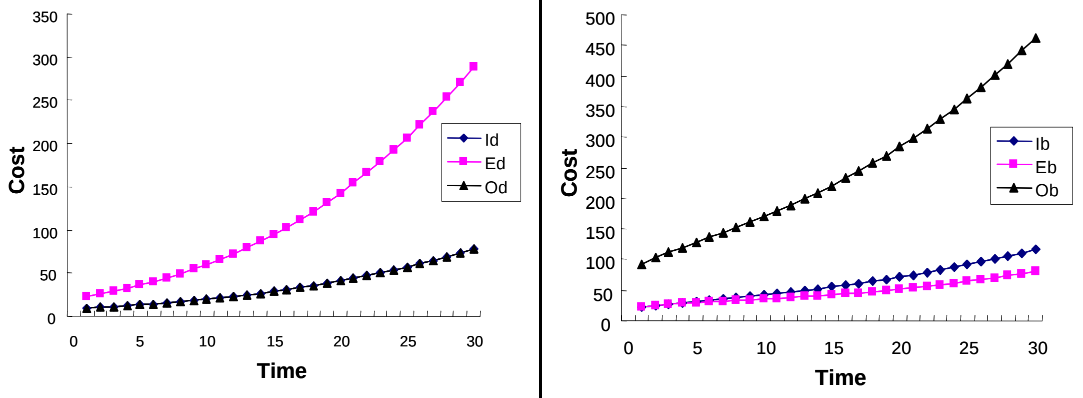
Proposition 9. *When the learning-by-doing process is more productive for the breadth of knowledge than the other two learning processes, a relatively larger amount of effort should be made at the early time and toward the end of the time period than in the middle of the time period in order to obtain the optimal level of accumulated knowledge.*

Interestingly, there exist some differences in optimal decisions on the remaining learning processes depending on which process is chosen to be more productive. When the learning-by-



(a) Cost curve when learning-by-investment process is more productive than the other two learning processes**

(b) Cost curve when learning-by-doing process is more productive for the breadth than the other two learning processes



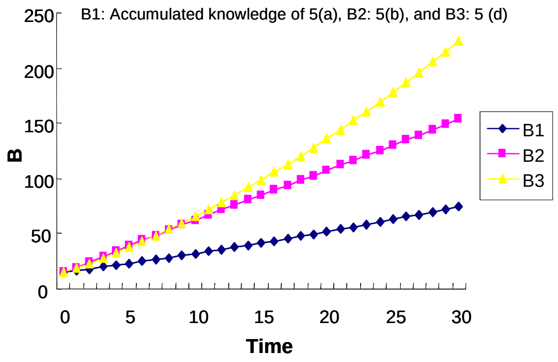
(c) Cost curve when learning-by-doing process is more productive for the depth than the other two learning processes

(d) Cost curve when learning-from-others process is more productive than the other two learning processes**

Figure 5. Impact of Productivity of One of Three Learning Processes on Other Processes

*Cost, here, indicates the general cost and it represents investment or effort for learning.

**The patterns of cost required are identical when “learning-by-investment” and “learning-from-others” processes are more productive for either the breadth or the depth of knowledge.



(e) Accumulated breadth of knowledge for different highly productive processes (as shown in 5(a), (b), and (d)

Figure 5. Impact of Productivity of One of Three Learning Processes on Other Processes (Continued)

doing process is more productive, the other less productive learning processes require an almost identical amount of resources (in Figure 5(b)). On the other hand, when the learning-by-investment process (in Figure 5(a)) or learning-from-others process (in Figure 5(d)) is more productive, the learning-by-doing process requires less resources than the other processes.

The learning-by-doing process is defined as the function of both resource and accumulated knowledge. The resources spent for learning are common across all three learning processes, whereas knowledge as an input that is accumulated in the past and grows over time is specific for the learning-by-doing process. Therefore, when the total resources spent are constant, less exogenous resources are likely spent for the learning-by-doing process than the others as shown in Figures 5(a) and 5(b). This result is caused by the difference in exponents of the learning processes. Even though exponents for both learning processes are set as an equal value, the actual exponent of the learning-by-doing process is always lower than the exponent of the learning-from-others process due to the property of logistic functions used for it. It follows that the productivity (exponent) of the learning process has a greater impact on the optimal decision on resource consumption than the additional input (accumulated knowledge in the learning-by-doing process and other members' knowledge in the learning-from-others process).

This phenomenon is more obvious when the learning-by-doing and learning-by-investment processes are compared in Figure 5(d). In this case, even though the learning-by-doing process has a more favorable condition than the learning-by-investment process, which has no additional input other than the investment, more resources are spent for the learning-by-investment process. This outcome verifies that the productivity of the learning process dominantly determines the amount of resources required for that process to obtain the optimal performance.

The impact of different exponents on the accumulated knowledge is represented in Figure 5(e) (B1, B2, and B3 correspond to the outcome of Figures 5(a), 5(b), and 5(d), respectively). This outcome shows that the greatest accumulated knowledge can be achieved when the learning-from-others process is more productive than the other processes. The worst performance happens when the learning-by-investment process is more productive than the other processes. This result implies the different impact of both productivity and additional input on the accumulated knowledge depending on the different learning process. Since the learning-from-others process utilizes two inputs (individual's effort and knowledge transfer from others), it produces a greater level of accumulated knowledge than the other learning processes when it is highly productive. Although the learning-by-doing process also has an additional input (previously accumulated knowledge) other than individual's effort, its productivity is lower than that of the learning-from-others process and it consequently generates less accumulated knowledge. Without any additional input other than investment, the worst performance is obtained when the learning-by-investment is more productive than the others in spite of the same overall productivity. These additional results are summarized in Propositions 10(a) and 10(b).

Proposition 10(a). *The productivity of a learning process has a greater impact on the optimal decision on investment or effort for learning than does the additional input of the learning process (knowledge accumulated in the past or knowledge transferred from other members).*

Proposition 10(b). *When the learning-by-investment process is more productive than the other two processes, the whole learning processes results in less accumulated knowledge than when the other processes are more productive.*

Discussion and Implications

The results show that the seven environmental conditions affect individual members' profit from the learning processes. Higher discount rate of cost, greater salvage value of total knowledge, and larger initial level of knowledge provide an individual member with information about the favorable conditions in which profit can be increased from the accumulated knowledge obtained during participation in an EIP. The member also achieves better performance from the learning processes, when more knowledge can be received from other members and the learning-by-doing process is more productive. On the other hand, if the member intends to acquire less durable knowledge during participation in the EIP, the profit is relatively low due to the high rate of knowledge obsolescence.

The results also indicate the importance of proper timing and amount of knowledge investment and effort. Under favorable conditions, individuals achieve a high level of performance when they maintain large investment and effort over time. If a user intends to acquire less durable knowledge that becomes obsolete rapidly, it is necessary to spend a small amount of resources evenly to catch up with market trends and thus achieve optimal performance. On the other hand, when the targeted knowledge is durable and sustains its market value over time, the optimal decision is to spend heavily for learning over time, and, in particular, make a large portion of investment and effort at a later point in time.

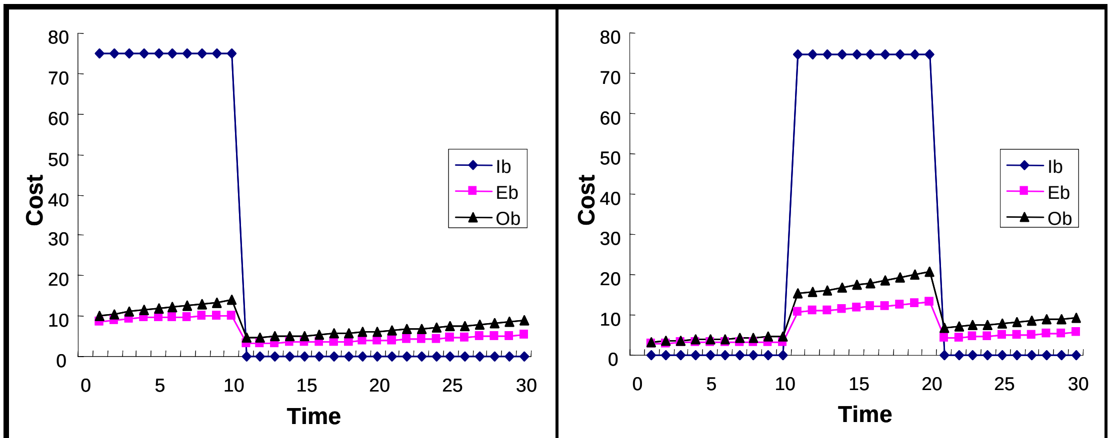
The findings of this research raise a couple of issues regarding the role of EIP in designing knowledge management systems. The first issue involves knowledge structure that is represented by division of labor developed in EIPs. According to Grant and Baden-Fuller (1995), it is important to obtain a congruence between product or service and knowledge domains for a firm to maximize the effectiveness in the exploitation of the firm's knowledge. Incongruence between them undermines competitive advantages as well as makes knowledge management efforts useless. As shown in the analysis, as the number of members increases, division of labor also increases and thus the accumulated knowledge of members is skewed toward depth, which may cause less motivation to invest in knowledge due to the narrow window of opportunity given to individuals. In this state, EIPs can be considered as designers of knowledge. It is important to balance the knowledge configuration utilized in an organization. This knowledge structure may provide members with information about the value of knowledge to be used as the investment criteria when they decide to invest in a specific area of knowledge. This may be critical to the competitive advantage of the firm (Cohen and Levinthal 1990).

The second issue concerns the mechanism for facilitating learning processes that inspire members to invest in knowledge and reduce confusion in interaction among members. Grant (1996) identified direction and routine as the primary mechanisms for knowledge acquisition. Direction refers to the rules, guidelines, and directives to be used for knowledge acquisition, which should be provided by EIPs. Organizational routines concern setting up patterns of interaction among members and achieving knowledge integration using the patterns. EIPs as a knowledge management system provide members with direction and organizational routines. These direction and organizational routines can be understood as (1) rules and (2) division of labor in the activity theory perspective (Kim et al. 2002). Rules include (1) the procedures facilitating the knowledge acquisition, (2) the reuse of a particular kind of knowledge, and (3) rewards for knowledge sharing. These rules embedded in EIPs help users exploit the acquisition of knowledge through learning-by-investment, learning-by-doing, and learning-from-others and maximize the performance of the systems by allowing various interactions among users to get the required knowledge. Hence, EIPs exert a lot of effort to develop rules pertinent to boosting the knowledge activities.

Division of labor in EIPs can be viewed in terms of a repository of highly specialized knowledge that can be shared by users in a multifunctional environment (Brown and Duguid 1998), as well as a set of actions or functions that facilitate knowledge activities that produce specialized knowledge. A strong tie between the repository and functions resolves the gap between doing and knowing, which in turn activates a feedback process for the learning-by-doing and learning-from-others processes. EIPs have to provide these two roles of division of labor simultaneously because the gap between knowing and doing should be minimized to maximize the effectiveness of knowledge management (Pfeffer and Sutton 1999). To this end, EIPs may introduce communities of practice where members performing similar tasks develop their own distinct criteria for knowledge evaluation (Pfeffer and Sutton 1999).

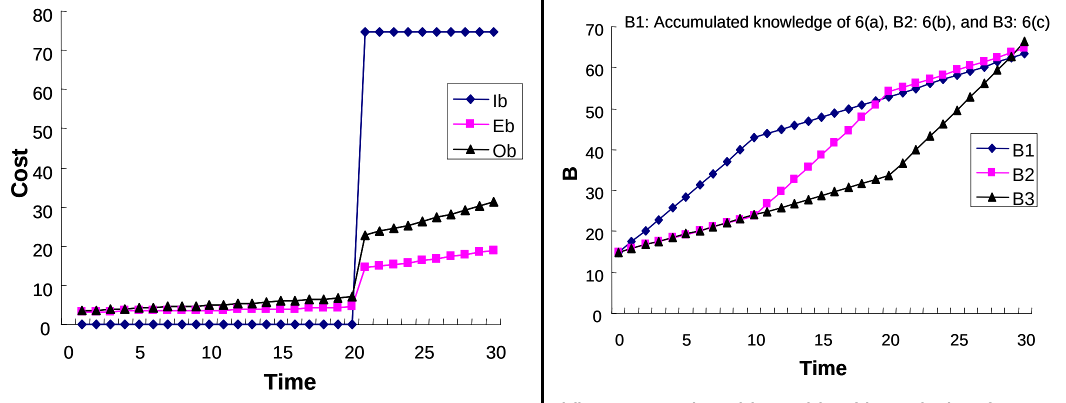
One more implication about division of labor can be obtained in the case that the EIP pursues balanced knowledge of each individual member. As discussed earlier, over time, each individual member possesses accumulated knowledge that improves in terms of only its depth due to the impact of division of labor, which may discourage individuals from investing in the breadth of knowledge. EIPs may encourage their members to increase their investments on the learning processes other than the learning-by-doing process to improve the breadth of knowledge by offering some incentives to individuals. This is possible because the productivity of the investment in learning-by-investment and learning-from-others processes for the breadth of knowledge are fixed but that of the investment in learning-by-doing process for the breadth of knowledge decreases over time. The large resource consumption for either the learning-by-investment processes or learning-from-others process necessarily accompanies the investment in the other learning processes in order for the optimality to be held. This phenomenon observed in Figures 6(a), 6(b), and 6(c) shows the result of the cases where resources are invested at different time periods in order to enhance the breadth of knowledge. Figure 6(d) illustrates the resultant accumulated knowledge (breadth) when the investment is concentrated between time 1 and time 10 (B1), between time 11 and time 20 (B2), and between time 21 and time 30 (B3).

As shown in Figure 6, when most resources are committed to a learning process for a limited time, the greater the resources spent on the other learning processes during the same time period than at other times. This accompanying effect among different learning processes implies one note of caution to EIPs, which intend to improve a certain attribute of knowledge by encouraging a particular learning process. When the EIP emphasizes one learning process, it is possible that the members make an unnecessarily heavy investment in the other processes, which is unproductive, due to the accompanying effect. Consequently, the incautious policy of the EIP that encourages its members to make a heavy invest-



(a) Cost curve for most investment in breadth of knowledge from time 1 to time 10

(b) Cost curve for most investment in breadth of knowledge from time 11 to time 20



(c) Cost curve for most investment in breadth of knowledge from time 21 to time 30

(d) Accumulated breadth of knowledge for different investment timings

Figure 6. Impact of Timing of Investment on Learning Processes

ment in one learning process can result in excessive resource consumption and biased knowledge accumulation. Hence, EIPs have to determine the tolerance limits of investments in less productive learning process when designing individual knowledge acquisition policy in terms of division labor.

The outcomes in Figures 6(a), 6(b), and 6(c) also imply the existence of a certain interdependency among the three learning processes. These graphs show that the change of investment amount and timing (i.e., investment pattern) in one learning process clearly affects the optimal investment patterns in the other learning processes. Figure 6(d) shows that the timing of heavy investment affects the total amount of accumulated knowledge as well as its rate. Due to the discount rate, when the heavy investment is made at the later point of time, the better the performance that is achieved. These results indicate that for individuals, the proper investment timing and correct decisions on the investment amount are important to maximize the benefits from knowledge investment. Investing in the correct learning process for the correct attribute of knowledge is also an important mission for individual members to maximize benefit from knowledge investment.

Conclusions

This research provides a comprehensive model of knowledge acquisition in an EIP, which describes three distinct types of learning processes: learning-by-investment, learning-by-doing, and learning-from-others. Cumulative knowledge resulting from the learning processes is measured in terms of two distinct attributes: depth and breadth of knowledge. The model is solved based on optimal control theory, and analyses of numerical examples are conducted by using a shooting procedure due to the complexity of the problem.

The current study is expected to contribute to the literature in two ways. First, this study identifies three major learning processes for individuals to obtain knowledge and seven environmental factors affecting the timing of individuals' investment decision making and the amount of the investment. In sum, EIP members need to spend a greater amount of resources to obtain the optimal level of knowledge under favorable conditions of high discount rate of cost, high salvage value of knowledge, large initial level of knowledge, large others' knowledge, and the productive learning process. It is also argued that it is preferable to delay a large portion of the total spending until a later point of the given time period. In addition, it is suggested that investment or effort should be selectively made for a more productive learning process, when differences in productivity exist among learning processes. Only the proper resource allocation depending on the productivity of the learning process results in the best performance in the knowledge acquisition process.

Second, this study raises several issues regarding the design of knowledge management and its systems in the context of EIPs. This study argues that EIPs have to take account of the level of congruence between product and knowledge domain when designing division of labor to make the system effective. The effective structure of knowledge in EIPs may provide valuable information and incentive for members to invest in required knowledge. This research also contends that to facilitate the knowledge acquisition process, EIPs incorporate rules for communications, reuse of knowledge, and rewards, and establish a tight link between functions and knowledge repositories to make sure knowing is directly connected to doing. This study also suggests the introduction of communities of practice to provide evidence and warrants so that individuals achieve the target knowledge without confusion. A critical review of the two issues addressed above is important for the success of an EIP as a knowledge management organization. Hence, these areas can be further studied to evaluate the effectiveness of EIPs.

This study has some limitations that are also opportunities for future research. First, the proposed model defines the learning process based on breadth and depth of knowledge. However, this research does not go further to handle operationalization of the concepts of breadth and depth of knowledge so as to provide a measurement model of knowledge. Future research is desired to address this issue of operationalization to capture the very nature of the attributes of knowledge. Second, the current model does not address the issue of incentives provided to members of the EIP to encourage them to transfer their knowledge to others. Offering incentives for promoting communication is quite important in the learning-from-others process, because each individual member of the EIP may be willing to transfer knowledge to other members only when direct benefits can be gained. Third, the original problem proposed by this study is formulated as a continuous time model to address realistic situations. However, a discrete time model may provide potential to allow sophisticated analyses. Fourth, this study uses numerical examples for analyzing the proposed learning model. We are following the literature that has focused on the development of simulation technologies for virtual representation of real world structures. This literature (Carley and Lin 1997; Raghu et al. 2004) constitutes an alternative and complementary path for knowledge modeling. The results of analyses in this study, however, may not fully overcome the natural limitation of example-based analysis, even though we have interpreted the results of analyses based on general trends of outcomes rather than the specific optimal solutions corresponding to the parameters that are arbitrarily chosen. In order to obtain robust results from this study, it is possible that the parameters need to be determined through empirical studies. Fifth, every result of this study is based on outcomes that are analyzed at the individual level. Even though we touch upon the issue of EIP designs, only the study done at the organizational level, which considers the overall knowledge accumulation made by various members, can provide the EIP manager with solid conclusions about managerial implications. Future studies may pursue this issue by considering the learning process of collective individual members to maximize their total knowledge accumulation. Finally, the model in this study represents the learning process as a Cobb-Douglas production function. Future studies can investigate possible complimentary or substitutionary relationships among investment, accumulated knowledge, and knowledge transferred from other members as major inputs of the whole knowledge creation system. In the future study, the constant elasticity substitution (CES) production function (Dorroh et al. 1994) can potentially be applied to investigate relationships among the three learning processes.

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Appendix A

Solutions for the Mathematical Model

The simplified optimization problem (Equations (8) through (14)) can be solved by applying the optimal control theory (Sethi and Thompson 1981). At first, the Hamiltonian equation is

$$\begin{aligned} H = & -\left(I_{bt} + I_{dt} + E_{bt} + E_{dt} + O_{bt} + O_{dt}\right)\cdot e^{-\rho\cdot t} + \varpi_{bt}\cdot\eta_b\cdot I_{bt}^{\alpha_b}\cdot\left(E_{bt}\cdot B_t\right)^{\beta_{bt}} \\ & \cdot\left(\theta_b\cdot O_{bt}\cdot Z_{bt}\right)^{\gamma_b} + \varpi_{dt}\cdot\eta_d\cdot I_{dt}^{\alpha_d}\cdot\left(E_{dt}\cdot D_t\right)^{\beta_{dt}}\cdot\left(\theta_d\cdot O_{dt}\cdot Z_{dt}\right)^{\gamma_d} \end{aligned} \tag{A-1}$$

In the given Hamiltonian equation, decisions for the breadth and the depth of knowledge (I_{bt}, I_{dt}, E_{bt}, E_{dt}, O_{bt}, and O_{dt}) are the control variables, and B_t and D_t are the state variables, which indicate the state of accumulated knowledge at time *t*. ϖ_{bt} and ϖ_{bt} serve as adjoint variables, and Z_{bt} and Z_{dt} are the exogenous variables.

The necessary conditions for optimality based on the optimal control theory are

$$\frac{\partial H}{\partial I_{bt}} = -e^{-\rho\cdot t} + \varpi_{bt}\cdot\eta_b\cdot\alpha_b\cdot I_{bt}^{\alpha_b - 1}\cdot\left(E_{bt}\cdot B_t\right)^{\beta_{bt}}\cdot\left(\theta_b\cdot O_{bt}\cdot Z_{bt}\right)^{\gamma_b} = 0 \tag{A-2}$$

$$\frac{\partial H}{\partial I_{dt}} = -e^{-\rho\cdot t} + \varpi_{dt}\cdot\eta_d\cdot\alpha_d\cdot I_{dt}^{\alpha_d - 1}\cdot\left(E_{dt}\cdot D_t\right)^{\beta_{dt}}\cdot\left(\theta_d\cdot O_{dt}\cdot Z_{dt}\right)^{\gamma_d} = 0 \tag{A-3}$$

$$\frac{\partial H}{\partial E_{bt}} = -e^{-\rho\cdot t} + \varpi_{bt}\cdot\eta_b\cdot I_{bt}^{\alpha_b}\cdot\beta_{bt}\cdot E_{bt}^{\beta_{bt} - 1}\cdot B^{\beta_{bt}}\cdot\left(\theta_b\cdot O_{bt}\cdot Z_{bt}\right)^{\gamma_b} = 0 \tag{A-4}$$

$$\frac{\partial H}{\partial E_{dt}} = -e^{-\rho\cdot t} + \varpi_{dt}\cdot\eta_d\cdot I_{dt}^{\alpha_d}\cdot\beta_{dt}\cdot E_{dt}^{\beta_{dt} - 1}\cdot D^{\beta_{dt}}\cdot\left(\theta_d\cdot O_{dt}\cdot Z_{dt}\right)^{\gamma_d} = 0 \tag{A-5}$$

$$\frac{\partial H}{\partial O_{bt}} = -e^{-\rho\cdot t} + \varpi_{bt}\cdot\eta_b\cdot I_{bt}^{\alpha_b}\cdot\left(E_{bt}\cdot B_t\right)^{\beta_{bt}}\cdot\gamma_b\cdot O_{bt}^{\gamma_b - 1}\cdot\left(\theta_b\cdot Z_{bt}\right)^{\gamma_b} = 0 \tag{A-6}$$

$$\frac{\partial H}{\partial O_{dt}} = -e^{-\rho\cdot t} + \varpi_{dt}\cdot\eta_d\cdot I_{dt}^{\alpha_d}\cdot\left(E_{dt}\cdot D_t\right)^{\beta_{dt}}\cdot\gamma_d\cdot O_{dt}^{\gamma_d - 1}\cdot\left(\theta_d\cdot Z_{dt}\right)^{\gamma_d} = 0 \tag{A-7}$$

$$\frac{\partial H}{\partial B_t} = \varpi_{bt}\cdot\eta_b\cdot I_{bt}^{\alpha_b}\cdot\beta_{bt}\cdot B_t^{\beta_{bt} - 1}\cdot E_{bt}^{\beta_{bt}}\cdot\left(\theta_b\cdot O_{bt}\cdot Z_{bt}\right)^{\gamma_b} = -\varpi'_{bt} \tag{A-8}$$

$$\frac{\partial H}{\partial D_t} = \varpi_{dt}\cdot\eta_d\cdot I_{dt}^{\alpha_d}\cdot\beta_{dt}\cdot D_t^{\beta_{dt} - 1}\cdot E_{dt}^{\beta_{dt}}\cdot\left(\theta_d\cdot O_{dt}\cdot Z_{dt}\right)^{\gamma_d} = -\varpi'_{dt} \tag{A-9}$$

$$\frac{\partial H}{\partial \varpi_{bt}} = \eta_b \cdot I_{bt}^{\alpha_b} \cdot \left(E_{bt} \cdot B_t\right)^{\beta_{bt}} \cdot \left(\theta_b \cdot O_{bt} \cdot Z_{bt}\right)^{\gamma_b} = b_t \tag{A-10}$$

$$\frac{\partial H}{\partial \varpi_{dt}} = \eta_d \cdot I_{dt}^{\alpha_d} \cdot \left(E_{dt} \cdot D_t\right)^{\beta_{dt}} \cdot \left(\theta_d \cdot O_{dt} \cdot Z_{dt}\right)^{\gamma_d} = d_t \tag{A-11}$$

Additional equations can be obtained from the salvage value of the objective function (8).

$$\varpi_{bT} = V \cdot D_T \cdot e^{-\sigma \cdot T} \tag{A-12}$$

$$\varpi_{dT} = V \cdot B_T \cdot e^{-\sigma \cdot T} \tag{A-13}$$

When Equations (A-8) and (A-12) are solved simultaneously, the following equation is obtained.

$$\varpi_{bt} = V \cdot D_T \cdot e^{\int_t^T \left[\eta_b \cdot I_{b\tau}^{\alpha_b} \cdot \beta_{b\tau} \cdot B_\tau^{\beta_{b\tau}-1} \cdot E_{b\tau}^{\beta_{b\tau}} \cdot (\theta_b \cdot O_{b\tau} \cdot Z_{b\tau})^{\gamma_b}\right] d\tau - \sigma \cdot T} \tag{A-14}$$

Equation (A-15) is also obtained by solving Equations (A-9) and (A-13) simultaneously.

$$\varpi_{dt} = V \cdot B_T \cdot e^{\int_t^T \left[\eta_d \cdot I_{d\tau}^{\alpha_d} \cdot \beta_{d\tau} \cdot D_\tau^{\beta_{d\tau}-1} \cdot E_{d\tau}^{\beta_{d\tau}} \cdot (\theta_d \cdot O_{d\tau} \cdot Z_{d\tau})^{\gamma_d}\right] d\tau - \sigma \cdot T} \tag{A-15}$$

By plugging Equation (A-14) into Equations (A-2), (A-4), and (A-6), Equations (A-16), (A-17), and (A-18) can be obtained.

$$\begin{aligned} &-e^{-\rho \cdot t} + \eta_b \cdot \alpha_b \cdot I_{bt}^{\alpha_b - 1} \cdot \left(E_{bt} \cdot B_t\right)^{\beta_{bt}} \cdot \left(\theta_b \cdot O_{bt} \cdot Z_{bt}\right)^{\gamma_b} \cdot V \\ &\cdot D_T \cdot e^{\int_t^T \left[\eta_b \cdot I_{b\tau}^{\alpha_b} \cdot \beta_{b\tau} \cdot B_\tau^{\beta_{b\tau}-1} \cdot E_{b\tau}^{\beta_{b\tau}} \cdot (\theta_b \cdot O_{b\tau} \cdot Z_{b\tau})^{\gamma_b}\right] d\tau - \sigma \cdot T} = 0 \end{aligned} \tag{A-16}$$

$$\begin{aligned} &-e^{-\rho \cdot t} + \eta_b \cdot I_{bt}^{\alpha_b} \cdot \beta_{bt} \cdot E_{bt}^{\beta_{bt}-1} \cdot B^{\beta_{bt}} \cdot \left(\theta_b \cdot O_{bt} \cdot Z_{bt}\right)^{\gamma_b} \cdot V \\ &\cdot D_T \cdot e^{\int_t^T \left[\eta_b \cdot I_{b\tau}^{\alpha_b} \cdot \beta_{b\tau} \cdot B_\tau^{\beta_{b\tau}-1} \cdot E_{b\tau}^{\beta_{b\tau}} \cdot (\theta_b \cdot O_{b\tau} \cdot Z_{b\tau})^{\gamma_b}\right] d\tau - \sigma \cdot T} = 0 \end{aligned} \tag{A-17}$$

$$\begin{aligned} &-e^{-\rho \cdot t} + \eta_b \cdot I_{bt}^{\alpha_b} \cdot \left(E_{bt} \cdot B_t\right)^{\beta_{bt}} \cdot \gamma_b \cdot O_{bt}^{\gamma_b - 1} \cdot \left(\theta_b \cdot Z_{bt}\right)^{\gamma_b} \\ &\cdot V \cdot D_T \cdot e^{\int_t^T \left[\eta_b \cdot I_{b\tau}^{\alpha_b} \cdot \beta_{b\tau} \cdot B_\tau^{\beta_{b\tau}-1} \cdot E_{b\tau}^{\beta_{b\tau}} \cdot (\theta_b \cdot O_{b\tau} \cdot Z_{b\tau})^{\gamma_b}\right] d\tau - \sigma \cdot T} = 0 \end{aligned} \tag{A-18}$$

Equations (A-19), (A-20), and (A-21) are obtained by inserting (A-15) into (A-3), (A-5), and (A-7), respectively.

$$\begin{aligned} &-e^{-\rho \cdot t} + \eta_d \cdot \alpha_d \cdot I_{dt}^{\alpha_d - 1} \cdot \left(E_{dt} \cdot D_t\right)^{\beta_{dt}} \cdot \left(\theta_d \cdot O_{dt} \cdot Z_{dt}\right)^{\gamma_d} \\ &\cdot V \cdot B_T \cdot e^{\int_t^T \left[\eta_d \cdot I_{d\tau}^{\alpha_d} \cdot \beta_{d\tau} \cdot D_\tau^{\beta_{d\tau}-1} \cdot E_{d\tau}^{\beta_{d\tau}} \cdot (\theta_d \cdot O_{d\tau} \cdot Z_{d\tau})^{\gamma_d}\right] d\tau - \sigma \cdot T} = 0 \end{aligned} \tag{A-19}$$

$$\begin{aligned}&-e^{-\rho\cdot t}+\eta_d\cdot I_{dt}^{\alpha_d}\cdot\beta_{dt}\cdot E_{dt}^{\beta_{dt}-1}\cdot D^{\beta_{dt}}\cdot\left(\theta_d\cdot O_{dt}\cdot Z_{dt}\right)^{\gamma_d}\\&\cdot V\cdot B_T\cdot e^{\int_t^T\left[\eta_d\cdot I_{d\tau}^{\alpha_d}\cdot\beta_{d\tau}\cdot D_\tau^{\beta_{d\tau}-1}\cdot E_{d\tau}^{\beta_{d\tau}}\cdot(\theta_d\cdot O_{d\tau}\cdot Z_{d\tau})^{\gamma_d}\right]d\tau-\sigma\cdot T}=0\end{aligned}\tag{A-20}$$

$$\begin{aligned}&-e^{-\rho\cdot t}+\eta_d\cdot I_{dt}^{\alpha_d}\cdot\left(E_{dt}\cdot D_t\right)^{\beta_{dt}}\cdot\gamma_d\cdot O_{dt}^{\gamma_d-1}\cdot\left(\theta_d\cdot Z_{dt}\right)^{\gamma_d}\\&\cdot V\cdot B_T\cdot e^{\int_t^T\left[\eta_d\cdot I_{d\tau}^{\alpha_d}\cdot\beta_{d\tau}\cdot D_\tau^{\beta_{d\tau}-1}\cdot E_{d\tau}^{\beta_{d\tau}}\cdot(\theta_d\cdot O_{d\tau}\cdot Z_{d\tau})^{\gamma_d}\right]d\tau-\sigma\cdot T}=0\end{aligned}\tag{A-21}$$

When Equations (A-10) and (13) are solved simultaneously, Equation (A-22) is obtained.

$$B_t=\underline{B}\cdot e^{\int_0^t\left[\eta_b\cdot I_{b\tau}^{\alpha_b}\cdot E_{b\tau}^{\beta_{b\tau}}\cdot B_\tau^{\beta_{b\tau}-1}\cdot(\theta_b\cdot O_{b\tau}\cdot Z_{b\tau})^{\gamma_b}\right]d\tau}\tag{A-22}$$

Equation (A-23) is also obtained by solving Equations (A-11) and (14) simultaneously.

$$D_t=\underline{D}\cdot e^{\int_0^t\left[\eta_d\cdot I_{d\tau}^{\alpha_d}\cdot E_{d\tau}^{\beta_{d\tau}}\cdot D_\tau^{\beta_{d\tau}-1}\cdot(\theta_d\cdot O_{d\tau}\cdot Z_{d\tau})^{\gamma_d}\right]d\tau}\tag{A-23}$$

Finally, the optimal solution of the optimization problem (I_{bt}, I_{dt}, E_{bt}, E_{dt}, O_{bt}, and O_{dt}) can be obtained by solving Equations (A-16), (A-17), (A-18), (A-19), (A-20), (A-21), (A-22), and (A-23) simultaneously. Since these eight equations are nonlinear, a closed form of solutions of I_{bt}, I_{dt}, E_{bt}, E_{dt}, O_{bt},, O_{dt}, B_t and D_t cannot be achieved in this problem. With the given specified parameters, the numerical optimal solution can be provided by applying the shooting procedure. Detailed steps of the shooting procedure are illustrated in the following:

Step 1. Set arbitrary initial values for old_sum_x1, old_sum_y1, old_sum_x2, and old_sum_y2.

Since the closed form of optimal solutions for I_{bt}, I_{dt}, E_{bt}, E_{dt}, O_{bt},, O_{dt}, B_t and D_t *are unknown, the integral portions of Equations (A-16), (A-17), (A-18), (A-19), (A-20), (A-21), (A-22), and (A-23) are approximated as the following discrete summation forms:*

$$old\_sum\_x1=\sum_{\tau=t+1}^{T}\left[\eta_b\cdot I_{b\tau}^{\alpha_b}\cdot\beta_{b\tau}\cdot B_\tau^{\beta_{b\tau}-1}\cdot E_{b\tau}^{\beta_{b\tau}}\cdot\left(\theta_b\cdot O_{b\tau}\cdot Z_{b\tau}\right)^{\gamma_b}\right]$$

$$old\_sum\_x2=\sum_{\tau=t+1}^{T}\left[\eta_d\cdot I_{d\tau}^{\alpha_d}\cdot\beta_{d\tau}\cdot D_\tau^{\beta_{d\tau}-1}\cdot E_{d\tau}^{\beta_{d\tau}}\cdot\left(\theta_d\cdot O_{d\tau}\cdot Z_{d\tau}\right)^{\gamma_d}\right]$$

$$old\_sum\_y1=\sum_{\tau=1}^{t}\left[\eta_b\cdot I_{b\tau}^{\alpha_b}\cdot E_{b\tau}^{\beta_{b\tau}}\cdot B_\tau^{\beta_{b\tau}-1}\cdot\left(\theta_b\cdot O_{b\tau}\cdot Z_{b\tau}\right)^{\gamma_b}\right]$$

$$old\_sum\_y2=\sum_{\tau=1}^{t}\left[\eta_d\cdot I_{d\tau}^{\alpha_d}\cdot E_{d\tau}^{\beta_{d\tau}}\cdot D_\tau^{\beta_{d\tau}-1}\cdot\left(\theta_d\cdot O_{d\tau}\cdot Z_{d\tau}\right)^{\gamma_d}\right]$$

Initial values of B_T *and* D_T *are obtained by using Equations (A-22) and (A-23) with values from old_sum_y1 and old_sum_y2.*

Step 2. Solve Equations (A-16), (A-17), (A-18), and (A-22) by applying a non-linear simultaneous equation system with the given values of old_sum_x1 and old_sum_y1.

Solve Equations (A-19), (A-20), (A-21), and (A-23) with old_sum_x2 and old_sum_ y2 in the same manner.

Step 3. Compute new values of old_sum_x1 (new_sum_x1), old_sum_y1 (new_sum_y1, old_sum_x2 (new_sum_x2), old_sum_y2 (new_sum_y2)) by using the values of I_{bt}, I_{dt}, E_{bt}, E_{dt}, O_{bt},, O_{dt}, B_t and D_t *obtained from Step 2.*

```
Step 4. If  |   new_sum_x1 - old_sum_x1 | > tol_x1,  | new_sum_y1 - old_sum_y1 | > tol_y1,
            |   new_sum_x2 - old_sum_x2 | > tol_x2, or  | new_sum_y2 - old_sum_y2 | > tol_y2
                    with certain small values of tolerance for tol_x1, tol_y1, tol_x2, and tol_y2
                set old_sum_x1 = new_sum_x1*ch+old_sum_x1*(1-ch)
                    old_sum_y1 = new_sum_y1*ch+old_sum_y1*(1-ch)
                    old_sum_x2 = new_sum_x2*ch+old_sum_x2*(1-ch)
                    old_sum_y2 = new_sum_y2*ch+old_sum_y2*(1-ch)
                    where 0 < ch < 1
                and obtain the new values of $B_T$ and $D_T$ by using  new values of old_sum_y1 and
                    old_sum_y2 in Equations (A-22) and (A-23)
                and then go to Step 2
            else exit.
```

By applying the shooting procedure to generate optimal solutions, this study uses numerical examples with combinations of various values of parameters.